MINRES/LS-5

5th Workshop on
Minimum Residual & Least-Squares Finite Element Methods

PONTIFICIA
UNIVERSIDAD
CATÓLICA
DE CHILE

Santiago, Chile

October 5 - 7, 2022

This document has been created on the base of
https://github.com/maximelucas/AMCOS_booklet.

Contents

About

For an up-to-date version of the program and further information visit the workshop web page
http://minres.mat.uc.cl
Document version: October 4, 2022

Welcome

The aim of this workshop is to bring together leading international researchers in the area of minimum residual and least-squares finite element methods. This is the fifth edition and takes place Oct. 5-7, 2022 at the Pontifical Catholic University of Chile in Santiago, Chile. The first workshop in this series took place 2013 in Austin, followed by the second workshop 2015 in Delft, the third one 2017 in Portland, and the fourth one 2019 in Berlin.

Scientific committee

Pavel Bochev	Sandia National Lab, USA
Zhiqiang Cai	Purdue University, USA
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Local assistant

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Timetable

Wednesday, Oct. 5

9:15–9:30	**Registration**	
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9:30–10:00	**L. Demkowicz**	DPG Progress at Oden
10:00–10:30	**J. Badger**	Scalable simulation of fiber laser model with DPG
10:30–11:00	**N. Roberts**	DPG for Vlasov: Two Formulations and Selected Results
11:00–11:30	**M. Łoś**	Least-squares space-time formulation for advection-diffusion problem with efficient linear solver based on matrix compression
11:30–12:15	**Coffee**	
12:15–13:45	**Session 2** (Chair: *M. Sánchez*)	
12:15–12:45	**M. Karkulik**	Space-time finite elements for the optimal control of parabolic equations
12:45–13:15	**G. Gantner**	Applications of a space-time FOSLS formulation for parabolic PDEs
13:15–13:45	**C. Wieners**	Space-time Discontinuous Galerkin methods and Discontinuous Petrov-Galerkin methods for hyperbolic linear Friedrichs systems
13:45–15:00	**Lunch**	
15:00–16:30	**Session 3** (Chair: *F. Fuentes*)	
15:00–15:30	**A. Chakraborty**	A continuous hp-mesh model for DPG finite element schemes with optimal test functions
15:30–16:00	**P. Vega**	An adaptive superconvergent finite element method based on local residual minimization
16:00–16:30	**J. Mora**	Using PolyDPG to simulate nonlinear mechanics of elastomers
16:30–17:00	**Coffee**	
17:00–18:00	**Session 4** (Chair: *I. Muga*)	
17:00–17:30	**F. Bertrand**	Approximation of eigenvalue problems with MINRES: Recent advances
17:30–18:00	**D. Boffi**	On the computation of Maxwell's eigenvalues with nodal elements
18:00–19:00	**Wine reception**	

Thursday, Oct. 6

9:30–11:30		**Session 5** (Chair: *M. Karkulik*)
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10:00–10:30	**F. Bersetche**	A deep first-order system least squares method for solving elliptic PDEs
10:30–11:00	**P. Sepúlveda**	A machine learning least-squares method with a weighted norm
11:00–11:30	**I. Muga**	Neural Control of Discrete Weak Formulations of PDEs
11:30–12:15		**Coffee**
12:15–13:45		**Session 6** (Chair: *J. Gopalakrishnan*)
12:15–12:45	**T. Führer**	MINRES for second-order PDEs with singular data
12:45–13:15	**S. Rojas**	Regularization of rough linear functionals and adaptivity
13:15–13:30	**P. Herrera**	A DPG method for the quad-div problem
13:30–13:45	**P. Bringmann**	Convergence analysis and numerical comparison of adaptive least-squares finite element methods
13:45–15:00		**Lunch**
15:00–16:00		**Session 7** (Chair: *P. Sepúlveda*)
15:00–15:30	**F. Fuentes**	Rigorous global minimization of nonlinear integral functionals using finite element discretizations and polynomial optimization
15:30–16:00	**K. Shi**	An L1 mixed DG method for second-order Elliptic Equations in the Non-divergence Form
16:00–16:30		**Coffee**
16:30–17:30		**Session 8** (Chair: *S. Rojas*)
16:30–17:00	**J. Muñoz-Matute**	DPG time-marching scheme with DPG semidiscretization is space for transient advection-reaction equations
17:00–17:30	**R. Stevenson**	Robust least-squares methods for the Helmholtz equation
20:00		**Dinner**

Friday, Oct. 7

10:00–11:30		**Session 9** (Chair: *T. Führer*)
10:00–10:30	**C. Carstensen**	Towards adaptive Hybrid high-order methods (HHO)
10:30–11:00	**A. Niemi**	DPG approach for dealing with stress concentrations
11:00–11:30	**N. Heuer**	DPG for Reissner-Mindlin plates, Part 2
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12:30		**Excursion**

Useful Information

Talks will be held in lecture hall **R28** situated on *Campus Oriente* of the *Pontificia Universidad Católica de Chile*. Signs will indicate the way to the lecture hall when you arrive at the campus (Jaime Guzmán Errázuriz 3300, Providencia).

Coffee is offered during designated breaks in front of the lecture hall.

Lunch is served during lunch break at the refectory.

WiFi is available through EDUROAM or alternatively, via the net UCeventos (password is published during the workshop).

The **conference dinner** will be held at the Restaurant **Park Lane** of the Hotel **Park Plaza** (Av. Ricardo Lyon 207, Providencia).

The **excursion** scheduled for Friday afternoon will take us to the *Matetic Vineyards*. The trip includes a tour of the winery and a wine tasting. A shuttle bus will take us there and departures approximately at 12:30 from the conference site. The bus will return approx. between 17:30 and 18:00 to the Hotel Park Plaza.

Transportation

Transfer from hotel to conference site

A shuttle bus from Hotel Park Plaza to the conference site and back is organized. On **Wednesday** and **Thursday** morning the bus to the conference site leaves at **8:40 am** from Hotel Park Plaza (waiting in the street Diego de Velasquez, just right from the hotel when looking at it). There is a map of the hotel location on the website `http://minres.mat.uc.cl`. On **Friday** morning the bus leaves at **9:00 am**.

Public transportation

Public transport in Santiago includes various subway and bus lines. It has to be paid with prepaid cards. A one way trip (including line changes) is about 800 CLP (less than 1 USD). You need to top your card before your trip. You can buy a card resp. top your card in any metro station. An overview of the transportation network can be found at `https://www.red.cl/en/`.

The metro stations closest to the hotels "Hotel Santiago Park Plaza", "Le Reve Boutique Hotel", and "Mr. Hoteles Providencia" are "Los Leones" and "Pedro de Valdivia", both Line 1.

The metro station closest to the workshop site is "Chile-España" (Line 3). Several bus lines have stops close to the workshop site. You can use the site `www.red.cl` (Plan a journey) to plan your trip with public transportation. Transportation between a place close to the hotels "Hotel Santiago Park Plaza", "Le Reve Boutique Hotel", and "Mr. Hoteles Providencia" and the workshop location will be provided.

Taxi

There are many taxis in Santiago which have to be paid in cash (local currency). Be aware that most of the drivers do not speak English. You can also use Uber (which requires internet connection).

Abstracts

List of Abstracts

DPG PROGRESS AT ODEN

LESZEK DEMKOWICZ*

ABSTRACT

I will give a progress report on three DPG related research subjects that we are currently pursuing in my group.

First, I will report on extensive numerical experiments for the *double adaptivity method* in context of 2D *confusion* problem and higher order elements. I will outline two different codes that we have built, the first one based on two independent data structures and the use of pointers, and the second one based on a single data structure but the use of weakly conforming elements that are not covered by the theory. This is a joint work with Jacob Salazar [1].

Then I will report on a stability and convergence analysis for acoustic and Maxwell waveguide problems and the *full envelope approximation*. The use of the exponential ansatz results in modified acoustics and Maxwell problems that are solved with the DPG method based on the ultraweak formulation. This is a joint work with Markus Melenk and Stefan Henneking [2].

Finally, I will report some preliminary numerical results on combining the DPG method with my old *automatic hp-adaptivity* scheme [3]. Utilizing the ultraweak DPG method, we replace the globally *hp*-refined grid, with an *hp*-refined grid based on DPG residual estimate, and the *Projection-Based (PB) interpolation* with just L^2-projections. This work is being done with Jonathan Zhang.

References

[1] Salazar, J. and Demkowicz, L., *The double adaptivity paradigm: conforming vs. weakly conforming test functions*, Oden Institute Report 2021/15, submitted.

[2] Melenk, M., Demkowicz, L. and Henneking, S., *Convergence of full envelope DPG method for waveguide problems*, in preparation.

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SCALABLE SIMULATION OF FIBER LASER MODEL WITH DPG

STEFAN HENNEKING, JACOB BADGER*, LESZEK DEMKOWICZ

ABSTRACT

Development of increasingly high-power fiber laser systems is of interest for a range of applications including manufacturing, medicine, and military defense. Power scaling of fiber lasers is limited by a complex trade-space of deleterious nonlinear effects including transverse mode instability (TMI)—a phenomenon in which interference of fiber modes induces an oscillatory thermal profile, resulting in chaotic transfer of energy between modes and degrading beam coherence. Modeling and simulation can provide valuable tools for exploring fiber designs that mitigate nonlinear effects; unfortunately, many simplified models fail to accurately predict high-power performance, explaining the need for high-fidelity modeling of fiber laser systems.

A coupled vectorial Maxwell and heat model under a DPG finite element discretization was recently used to simulate TMI in a step-index continuous-wave active-gain fiber laser amplifier in [1]; however, the computational expense of resolving highly-oscillatory electromagnetic fields limited the scale of simulation to < 1 cm of fiber. In the present work, the Maxwell model is supplanted by an *equivalent* vectorial envelope model that significantly eases discretization requirements; enabling three-dimensional simulation of TMI in full-length fibers with $> 1\,000\,000$ optical wavelengths. This model is currently limited to fibers with simple cross-sections due to relatively poor scaling of available solvers under transverse refinement; we thus detail progress on a distributed implementation of a scalable DPG multigrid solver [3] to enable simulation of more complex fiber geometries, including fiber bending effects.

The fiber laser model and DPG-MG solver are implemented in *hp*3D[1]—an open-source scalable *hp*-adaptive finite element software [2].

[1] https://github.com/Oden-EAG/hp3d

References

[1] Henneking, S., *A scalable hp-adaptive finite element software with applications in fiber optics*, Ph.D. thesis, The University of Texas at Austin (2021).

[2] Henneking, S. and Demkowicz, L., *hp3D User Manual*, arXiv:2207.12211, Jun. 2022.

[3] Petrides, S. and Demkowicz, L., *An adaptive multigrid solver for DPG methods with applications in linear acoustics and electromagnetics*, Comput. Math. Appl. 87 (2021), pp. 1999–2017.

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DPG FOR VLASOV: TWO FORMULATIONS AND SELECTED RESULTS

NATHAN V. ROBERTS, STEPHEN D. BOND, AND ERIC C. CYR

ABSTRACT

Efficient solution of the Vlasov equation, which can be up to six-dimensional, is key to the simulation of many difficult problems in plasma physics. The discontinuous Petrov-Galerkin (DPG) finite element methodology provides a framework for the development of stable (in the sense of LBB conditions) finite element formulations, with built-in mechanisms for adaptivity. We present two DPG-based formulations for Vlasov: a time-marching, backward-Euler formulation, and a space-time formulation, with an ultimate target of solving problems in the full seven-dimensional setting. For this purpose, we employ tensor-product data representations supported by recent additions to the Intrepid2 package within Trilinos, as well as corresponding developments within Camellia, a finite element library designed to facilitate rapid development of computationally efficient, hp-adaptive finite element solvers, starting with support for DPG. In this talk, we discuss our progress to date, including adaptive results from 1D1V time-marching and space-time Vlasov-Poisson problems.

References

[1] Michael A. Heroux, Roscoe A. Bartlett, Vicki E. Howle, Robert J. Hoekstra, Jonathan J. Hu, Tamara G. Kolda, Richard B. Lehoucq, Kevin R. Long, Roger P. Pawlowski, Eric T. Phipps, Andrew G. Salinger, Heidi K. Thornquist, Ray S. Tuminaro, James M. Willenbring, Alan Williams, and Kendall S. Stanley. An Overview of the Trilinos Project. *ACM Trans. Math. Softw.* (2005), 397-423. doi.org/10.1145/1089014.1089021.

[2] Nathan V. Roberts. Camellia: A Rapid Development Framework for Finite Element Solvers. *Computational Methods in Applied Mathematics* (2019). doi.org/10.1515/cmam-2018-0218.

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LEAST-SQUARES SPACE-TIME FORMULATION FOR ADVECTION-DIFFUSION PROBLEM WITH EFFICIENT LINEAR SOLVER BASED ON MATRIX COMPRESSION

MARCIN ŁOŚ*, MACIEJ PASZYŃSKI, MATEUSZ DOBIJA, ANNA PASZYŃSKA, PAULINA SEPÚLVEDA SALAS

ABSTRACT

We present a space-time formulation of the non-stationary advection-dominated advection-diffusion problem based on a constrained least-squares approach. For discretization we use Isogeometric Analysis and employ B-spline basis functions. While using standard separate time and space discretization, in some restricted cases it is possible to construct highly efficient linear solvers exploiting the structure of the problem and the time stepping schemes (e.g. [1]), with space-time formulation the resulting matrix has a more complex structure. We present the idea of an iterative solver employing a matrix compression technique based on recursive decomposition and singular value decomposition (SVD).

This work is supported by The European Union's Horizon 2020 Research and Innovation Program of the Marie Skłodowska-Curie grant agreement No. 777778, MATHROCKs.

References

[1] Behnoudfar, P., Calo, V.M., Łoś, M., Maczuga, P., Paszyński, M., *Variational splitting of high-order linear multistep methods for heat transfer and advection–diffusion parabolic problems*, Journal of Computational Science 63 (2022), pp. 1-11.

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SPACE-TIME FINITE ELEMENTS FOR THE OPTIMAL CONTROL OF PARABOLIC EQUATIONS

THOMAS FÜHRER, MICHAEL KARKULIK*

ABSTRACT

Recently, [1, 3] introduced space-time finite element methods for parabolic equations which are robust on space-time locally refined meshes and also easy to implement. In this talk, we show how to apply this approach to the optimal control of parabolic equations, cf. [2]. We give a short introduction on optimal control of PDE and point out the inherent problems when discretizing optimal control problems of parabolic equations with classical time-stepping methods. Then, we proceed with an a-priori as well as a-posteriori analysis of our new method. Finally, we conclude with some numerical experiments.

References

[1] T. Führer, M. Karkulik. *Space-time least-squares finite elements for parabolic equations*, Comput. Math. Appl., 92 (2021).

[2] T. Führer, M. Karkulik. *Space-time finite element methods for parabolic distributed optimal control problems* , arXiv:2208.09879, (2022).

[3] G. Gantner, R. Stevenson. *Further results on a space-time FOSLS formulation of parabolic PDEs*, ESAIM Math. Model. Numer. Anal. 55.1 (2021).

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APPLICATIONS OF A SPACE-TIME FOSLS FORMULATION FOR PARABOLIC PDES

GREGOR GANTNER*, ROB STEVENSON

ABSTRACT

While the common space-time variational formulation of a parabolic equation results in a bilinear form that is non-coercive, [1] recently proved well-posedness of a space-time first-order system least-squares (FOSLS) formulation of the heat equation, which corresponds to a symmetric and coercive bilinear form. In particular, the Galerkin approximation from any conforming trial space exists and is a quasi-best approximation. Additionally, the least-squares functional automatically provides a reliable and efficient error estimator. In [2], we have generalized the least-squares method of [1] to general second-order parabolic PDEs with possibly inhomogenoeus Dirichlet or Neumann boundary conditions. For homogeneous Dirichlet conditions, we present convergence of a standard adaptive finite element method driven by the least-squares estimator [2]. The convergence analysis is applicable to a wide range of least-squares formulations for other PDEs, answering a long-standing open question in the literature. Moreover, we employ the space-time least-squares method for parameter-dependent problems as well as optimal control problems [3]. In both cases, coercivity of the corresponding bilinear form plays a crucial role. Optimal control problems have been considered in parallel by [4].

References

[1] T. Führer and M. Karkulik., *Space–time least-squares finite elements for parabolic equations*, Comput. Math. Appl. 92 (2021), 27–36.

[2] G. Gantner and R. Stevenson, *Further results on a space-time FOSLS formulation of parabolic PDEs*, ESAIM Math. Model. Numer. Anal. 55.1 (2021), 283–299.

[3] G. Gantner and R. Stevenson, *Applications of a space-time FOSLS formulation for parabolic PDEs*, Preprint, arXiv:2208.09616 (2022).

[4] T. Führer and M. Karkulik., *Space-time finite element methods for parabolic distributed optimal control problems*, Preprint, arXiv:2208.09879 (2022).

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SPACE-TIME DISCONTINUOUS GALERKIN METHODS AND DISCONTINUOUS PETROV-GALERKIN METHODS FOR HYPERBOLIC LINEAR FRIEDRICHS SYSTEMS

CHRISTIAN WIENERS*

ABSTRACT

We study weak solutions and its approximation of hyperbolic linear symmetric first-order Friedrichs systems describing acoustic, elastic, or electro-magnetic waves. A DGP method for acoustics is considered in [2, 3]; convergence for the ideal DPG method is analyzed in the graph norm. For a discontinuous Galerkin discretization with full upwind in space and time, inf-sup stability and convergence estimates are provided with respect to a mesh-dependent DG norm in [1]; this relaxes the regularity assumptions required in the graph norm and improves the estimates the by a factor $h^{1/2}$ in the space-time cylinder.

In this talk we show that the results for the space-time DG method in the mesh-dependent DG norm transfer to Petrov–Galerkin methods by constructing suitable discrete Fortin operators which extend the inf-sup stability of the mesh-dependent DG method to inf-sup stability of the Petrov–Galerkin method. This provides convergence estimates for different variations of discontinuous Petrov-Galerkin methods, where the trace spaces are discontinuous on the space-time skeleton.

References

[1] D. Corallo, W. Dörfler, and C. Wieners. Space-time discontinuous Galerkin methods for weak solutions of hyperbolic linear symmetric Friedrichs systems. CRC 1173 Preprint 2022/36, Karlsruhe Institute of Technology, 2022.

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[3] J. Ernesti and C. Wieners. Space-time discontinuous Petrov-Galerkin methods for linear wave equations in heterogeneous media. *Computational Methods in Applied Mathematics*, 19(3):465–481, 2019.

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A CONTINUOUS hp–MESH MODEL FOR DPG FINITE ELEMENT SCHEMES WITH OPTIMAL TEST FUNCTIONS

ANKIT CHAKRABORTY*, GEORG MAY

ABSTRACT

In many industrial and academic applications, certain quantities of interest, such as flux across a specific boundary or solution in a particular sub-domain, are subject to more interest than the solution variable itself. In these cases, adapting the mesh for resolving the governing PDE's solution features may result in an unwanted increase in the number of degrees of freedom. In this context, goal-oriented mesh adaptation techniques have been critical for producing meshes that only focus on resolving the target functional. Typically, these adaptation techniques often compute the element size distribution by solving a compatible dual problem. However, this can be complemented by selecting a correct local polynomial order for approximating the primal variables.

In terms of meshing techniques, it has already been shown that metric-based mesh generation can produce anisotropic meshes having substantial advantage while resolving anisotropic flow features such as sharp boundary layers and singularities [2]. In this work, we present a goal-oriented metric-based mesh adaptation scheme where we employ the recently proposed DPG-star method for solving the compatible dual problem and the associated a posteriori error estimate for computing element size distribution. Also, we solve certain local problems and utilize the well-established energy norm error estimator [1] to obtain an appropriate polynomial order of approximation for the primal variables and anisotropy of the elements in the mesh.

References

[1] Leszek Demkowicz and Jay Gopalakrishnan,*A class of discontinuous Petrov–Galerkin methods. II. Optimal test functions*,Numerical Methods for Partial Differential Equations.2011

[2] Ankit Chakraborty, Ajay Mandyam Rangarajan, Georg May, *An anisotropic h-adaptive strategy for discontinuous Petrov-Galerkin schemes using a continuous mesh model*, Computers & Mathematics with Application. 2022

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AN ADAPTIVE SUPERCONVERGENT FINITE ELEMENT METHOD BASED ON LOCAL RESIDUAL MINIMIZATION

IGNACIO MUGA, SERGIO ROJAS, PATRICK VEGA*

ABSTRACT

During the last decades, residual minimization methods has been increasing in popularity due to its stabilization properties. Among the most popular is the discontinuous Petrov-Galerkin (dPG) method, introduced in [1]. In 2020, a new residual based Adaptive Stabilized Finite Element Method (AS-FEM) (see [2]) was introduced, combining a residual minimization approach with the inf-sup stability offered by a large class of discontinuous Galerkin (dG) methods. As in dPG methods, this method also delivers a stable solution and a residual representative. Inspired in [2], in this talk we will introduce a novel adaptive stabilized finite element method for a class of mixed methods. The method consists of performing a residual minimization in terms of the Stenberg's prostpocessing strategy (see [3]), being a superconvergent and fully localizable postprocess for the scalar variable. As a result, we obtain both, a superconvergent approximation for the scalar variable, and a residual representative to drive the adaptivity. However, the new scheme inherits the fully localizable property of Stenberg's postprocessing, implying that the cost of solving the residual minimization can be neglected, making it competitive with respect to standard a posteriori residual estimators. We will detail its derivation and will show its performance considering challenging diffusion problems.

References

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[2] Calo, V. M., Ern, A., Muga, I., and Rojas, S., *An adaptive stabilized conforming finite element method via residual minimization on dual discontinuous Galerkin norms*, CMAME 363 (2020), 112891.

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USING POLYDPG TO SIMULATE NONLINEAR MECHANICS OF ELASTOMERS

JAIME MORA-PAZ*, LESZEK DEMKOWICZ

ABSTRACT

The discontinuous Petrov-Galerkin (DPG) finite element methodology is known to grant discrete stability for any well-posed variational problem. The use of broken test spaces and the ultraweak variational formulation have let DPG be applied on meshes of general polytopal elements, a version of the method that we have labeled PolyDPG (see [1, 2] for theory and numerics in 2D and 3D). According to the elasticity models and numerical results developed in the 2020 PhD dissertation [3], the 3D version of PolyDPG is capable of simulating large compressive deformation of elastomeric foams that are modeled with hyperelastic constitutive relations, far overcoming the simulated stretches attained with traditional finite elements. The proposed approach for this kind of problem results in a better capturing of local deformations and stresses, along with the formerly observed capacity of simulating large global deformations.

References

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[2] Bacuta, C., Demkowicz, L., Mora Paz, J. and Xenophontos, Ch., *Analysis of non-conforming DPG methods on polyhedral meshes using fractional Sobolev norms*, Comput. Math. Appl., 91 (2021), pp. 215–241.

[3] Mora Paz, J., *PolyDPG: a Discontinuous Petrov-Galerkin methodology for polytopal meshes with applications to elasticity*, The University of Texas at Austin, PhD Thesis (2020).

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APPROXIMATION OF EIGENVALUE PROBLEMS WITH MINRES: RECENT ADVANCES

FLEURIANNE BERTRAND⋆

ABSTRACT

Accurate flux approximations are of interest in many applications and minimum residual methods involves the flux and the stress as independent variables approximated in a suitable H(div)-conforming finite element spaces. Considering the corresponding spectral problems is helpful to determine the response of materials to a given phenomenon and is crucial to our description of the world. In this talk, we therefore discuss recent advances concerning the spectral properties of operators associated with the corresponding least-squares and discontinuous Petrov-Galerkin finite-element minimization of the residual.

References

[1] FB and Daniele Boffi, *First order least-squares formulations for eigenvalue problems*, IMA Journal of Numerical Analysis, 42(2):1339-1363, 2021.

[2] FB and Daniele Boffi, *Least-squares formulations for eigenvalue problems associated with linear elasticity*, Computers and mathematics with applications, 95:19–27, 2021.

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ON THE COMPUTATION OF MAXWELL'S EIGENVALUES WITH NODAL ELEMENTS

DANIELE BOFFI*

ABSTRACT

We consider the finite element approximation of the eigenvalues and eigenfunctions of the resonant cavity associated with Maxwell's equation.

It is well known that with a standard Galerkin formulation the optimal convergence is achieved when edge elements are used [4].

Recent results on the approximation of the spectrum associated with finite element least squares formulations [3, 1, 2] can be extended to the Maxwell eigenvalue problem. This is straightforward in two dimensions and more elaborate in three dimensions.

One might wonder if such results are also valid when nodal elements are used for the approximation of the electric field.

The aim of this talk is to give an answer to this question and to compare the numerical results with other schemes involving nodal elements such as [5].

References

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[2] Alzaben, L., Bertrand, F., and Boffi, D., *On the spectrum of the finite element approximation of a three field formulation for linear elasticity*. Examples and Counterexamples 2 (2022), 100076.

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LEAST-SQUARES NEURAL NETWORK (LSNN) METHOD FOR HYPERBOLIC CONSERVATION LAWS

ZHIQIANG CAI*, JINGSHUANG CHEN, MIN LIU

ABSTRACT

Solutions of nonlinear hyperbolic conservation laws (HCLs) are often discontinuous due to shock formation; moreover, locations of shocks are *a priori* unknown. This presents a great challenge for traditional numerical methods because most of them are based on continuous or discontinuous piecewise polynomials on fixed meshes.

As an alternative, by employing a new class of approximating functions, *neural network* (NN), recently we proposed the least-squares neural network (LSNN) method for solving HCLs. The LSNN method shows a great potential to sharply capture shock without oscillation or smearing; moreover, its degrees of freedom are much less than those of mesh-based methods. Nevertheless, current iterative solvers for the LSNN discretization are computationally intensive and complicated.

In this talk, I will present our recent work [1, 2, 3] on the LSNN for solving linear and nonlinear scalar HCLs.

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A DEEP FIRST-ORDER SYSTEM LEAST SQUARES METHOD FOR SOLVING ELLIPTIC PDES

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ABSTRACT

We propose a First-Order System Least Squares (FOSLS) method based on deep-learning for numerically solving second-order elliptic PDEs. The method we propose is capable of dealing with either variational and non-variational problems, and because of its meshless nature, it can also deal with problems posed in high-dimensional domains. We prove the Γ-convergence of the neural network approximation towards the solution of the continuous problem, and extend the convergence proof to some well-known related methods. Finally, we present several numerical examples illustrating the performance of our discretization.

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A MACHINE LEARNING LEAST-SQUARES METHOD WITH A WEIGHTED NORM.

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ABSTRACT

A study of Neural Networks in combination with Finite Elements to obtain approximations of parametric PDEs is presented. This study is motivated by the works presented in [1] and [2]. The approach is to obtain a least-squares formulation with a discontinuous test space, endowed with a weighted inner product given by an artificial neural network. The block structure of the discrete approximation associated with the test inner product makes the computations easier to implement. Then, we train the modified scheme with a learning procedure and use a loss function that minimizes the quantity of interest. The potential of using Neural Networks for a parametric equation will be presented using different quantities of interest and loss functions.

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NEURAL CONTROL OF DISCRETE WEAK FORMULATIONS OF PDES

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ABSTRACT

We introduce the concept of neural control of discrete weak formulations of Partial Differential Equations (PDEs), in which finite element discretizations are intervened by using neural-network weight functions. The weight functions act as control variables that –through the minimization of a cost (or loss) functional– produce discrete solutions incorporating user-defined desirable attributes (e.g., known-data features, remotion of spurious oscillations, or precision at a certain quantities of interest).

Well-posedness and convergence of the cost-minimization problem are analyzed. In particular, we prove under certain conditions, that the discrete weak forms are stable, and that quasi-minimizing neural controls exist, which converge quasi-optimally. We specialize our analysis into Galerkin, least-squares, and minimal-residual formulations. Elementary numerical experiments support our findings and demonstrate the potential of the framework.

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MINRES FOR SECOND-ORDER PDES WITH SINGULAR DATA

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ABSTRACT

In this talk I present recent results on minimum residual methods (MINRES) for problems with singular data. Minimum residual methods such as the least-squares finite element method (FEM) or the discontinuous Petrov-Galerkin method with optimal test functions (DPG) usually exclude singular data, e.g., non square-integrable loads. We consider a DPG method and a least-squares FEM for the Poisson problem. For both methods we analyze regularization approaches that allow the use of singular load functionals, and also study the case of point loads. For all cases we prove appropriate convergence orders and present various numerical experiments that confirm our theoretical results.

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REGULARIZATION OF ROUGH LINEAR FUNCTIONALS AND ADAPTIVITY

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ABSTRACT

Rough linear functionals, such as Dirac Delta distributions, often appear on the right-hand side of variational formulations of PDEs. As they live in negative Sobolev spaces, they dramatically affect adaptive finite element procedures to approximate the solution of a given PDE. To overcome this drawback, we propose an alternative that, in a first step, computes a projection of the rough functional over piecewise polynomial spaces, up to a desired precision in a negative norm sense. The projection, being L^p-regular, is then used as the right-hand side of a regularized problem for which adaptive Galerkin methods performs better. An error analysis of the proposed methodology will be shown, together with numerical experiments.

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A DPG METHOD FOR THE QUAD-DIV PROBLEM

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ABSTRACT

The Quad-Div problem is related to the Quad-Curl problem in two dimensions. This kind of problems arise in several engineering and science problems, such as magneto-hydrodynamics, linear elasticity and inverse scattering theory [1].

In this talk we discuss the *Discontinous Petrov-Galerkin method with optimal test functions* (DPG method) [2] for the quad-div problem in a bounded Lipschitz polyhedral domain. The DPG method is a minimum residual method and is automatically stable. We develop an ultraweak formulation of a second-order reformulation. We prove its well-possedness in two and three dimensions. Then we construct a Fortin operator for $H(\nabla \mathrm{div})$ space and employ the DPG methodology that yields a quasi-optimal convergent numerical squeme. Finally, we show numerical experiments that confirms our theoretical results.

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CONVERGENCE ANALYSIS AND NUMERICAL COMPARISON OF ADAPTIVE LEAST-SQUARES FINITE ELEMENT METHODS

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ABSTRACT

Due to the built-in a posteriori error control, the least-squares finite element methods (LSFEMs) are a favourable choice for adaptive mesh-refining algorithms. Convergence results have been established for various adaptive LSFEMs in the literature. First, the built-in error estimator leads to Q-linear convergence in an adaptive algorithm with collective marking [4]. Second, an alternative residual-based error estimator and a separate marking strategy with data approximation even guarantee optimal convergence rates for the error in the natural underlying norm [2]. Third, collective marking with the alternative error estimator provides optimal convergence rates in a weaker norm [3]. An experimental comparison of all three adaptive algorithms confirms these findings [1]. The first part of this talk outlines the state-of-the-art for the convergence analysis of adaptive LSFEMs. The second part investigates the choice of the parameters in the marking and refinement strategies as well as the performance of the adaptive algorithms.

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RIGOROUS GLOBAL MINIMIZATION OF NONLINEAR INTEGRAL FUNCTIONALS USING FINITE ELEMENT DISCRETIZATIONS AND POLYNOMIAL OPTIMIZATION

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ABSTRACT

Computation of minima of nonlinear integral functionals (e.g. strain energy) is typically done by using gradient descent methods or some version of Newton's method on the Euler-Lagrange PDEs associated with the functional. These procedures only guarantee finding an approximation to a local minimum, but say nothing of whether the solution is a global minimum of the functional, which is often the goal. Finding an algorithm that provably converges to a global minimum and corresponding minimizer is a classical and fundamental challenge in many fields, including nonlinear elasticity, fluid mechanics, pattern formation and PDE analysis. In this work, we leverage theoretical tools from the fields of sparse polynomial optimization (within algebraic geometry) and finite element (FE) methods to present such an algorithm. The techniques include exploiting properties of sparse sum-of-squares (SOS) relaxations and Gamma convergence to prove convergence to a global minimum of a functional with an integrand with polynomial nonlinearities as the mesh is refined and the moment-SOS relaxation order is raised. We present numerical examples which result in excellent approximations to the global minima of different nonlinear functionals, including the pattern-forming Swift-Hohenberg free energy in two spatial dimensions.

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AN L^1 MIXED DG METHOD FOR SECOND-ORDER ELLIPTIC EQUATIONS IN THE NON-DIVERGENCE FORM

WEIFENG QIU, JIN REN, KE SHI* AND YUESHENG XU

ABSTRACT

In this talk we present an L^1 mixed DG method for second-order elliptic equations in the non-divergence form. The elliptic PDE in non-divergence form arises in the linearization of fully nonlinear PDEs. Due to the nature of the equations, classical finite element methods based on variational forms can not be employed directly. In this work, we propose a new optimization based finite element method which combines the classical DG framework with recently developed L^1 optimization technique. Convergence analysis in both energy norm and L^∞ norm are obtained under weak regularity assumption of the PDE (H^1). Such L^1 optimization problems are nondifferentiable and invalidate traditional graidnet methods. To overcome this difficulty, we characterize solutions of L^1 optimization as fixed-points of proximity equations and utilize matrix splitting technique to obtain a class of fixed-point proximity algorithms with convergence analysis. In addition, various numerical examples will be displayed to validate the analysis in the end.

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DPG TIME-MARCHING SCHEME WITH DPG SEMIDISCRETIZATION IS SPACE FOR TRANSIENT ADVECTION-REACTION EQUATIONS

J. MUÑOZ-MATUTE*, L. DEMKOWICZ, N. V. ROBERTS

ABSTRACT

We present a general methodology [1] to combine a Discontinuous Petrov-Galerkin (DPG) semidiscretization in space together with the recently developed DPG-based time-marching scheme [2, 3, 4] for transient advection-reaction problems. Regarding the semidiscretization in space with DPG we redefine the ideas of optimal testing and practicality of the method in this context. As the DPG-based time-marching scheme is of exponential-type, we also discuss how to efficiently compute the action of the exponential over vectors of the matrix coming from the space semidiscretization without assembling the full matrix. Finally, we verify the proposed method for 1D+time advection-reaction problems showing optimal convergence rates both in space and time for smooth solutions and more stable results for linear conservation laws comparing to the classical exponential integrators. The method we propose is practical in the computational sense and it can easily be generalized to higher dimensions and to other problems.

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ROBUST LEAST-SQUARES METHODS FOR THE HELMHOLTZ EQUATION

HARALD MONSUUR, ROB STEVENSON*

ABSTRACT

Inspired by [1], we present a well-posed ultra-weak first order system formulation of the Helmholtz equation with possibly inhomogeneous mixed Dirichlet, Neumann and Robin boundary conditions. By employing the optimal test-norm, least-squares discretizations yield the best approximation of the solution in the L_2-norm from the trial space. We present numerical results for the corresponding 'practical' method, as well as for an "LL^*"-method.

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TOWARDS ADAPTIVE HYBRID HIGH-ORDER METHODS (HHO)

CARSTEN CARSTENSEN*, SOPHIE PUTTKAMMER, NGOC TIEN TRAN

ABSTRACT

The novel methodology of skeletal schemes led to a new generation of nonstandard discretisations and HHO is one of many of those besides HDG, VEM, DPG, ... that generalize naturally to nonlinear problems. Can a variational crime lead to discretisations superior to conforming ones? The key for the success of higher-order schemes is through adaptive mesh-refining and the basis of this is a reliable and efficient a posteriori error analysis. The later is a topic in its infancy at least for HHO [5]. While over-stabilization enables some progress for DG and VEM, it is a refined analysis [6] that makes a stabilization-free a posteriori error estimate possible for the HHO [1]. The presentation reports on recent progress for linear problems [1] and then focusses on two very different nonlinear applications with — in comparison to conforming FEM — complementary advantages.

The fine-tuned extra-stabilized direct computation of guaranteed lower eigenvalue bounds allows for optimal convergence rates of a variant of HHO [2]. The appealing robust parameter selection allows the adaptive computation with higher convergence rates in numerical benchmarks.

The class of degenerate convex minimization problems with two-sided growth conditions and an appropriate convexity control [3, 4] allows convergent adaptive mesh-refinement for the dual stress-type variable.

The presentation reports on the state of research in joint projects with S. Puttkammer (Berlin) and N.T. Tran (Jena)

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DPG APPROACH FOR DEALING WITH STRESS CONCENTRATIONS

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ABSTRACT

Stress concentrations may occur in structural engineering e.g. in the vicinity of concentrated loads, abrupt transitions between materials or sharp re-entrant corners. Whether the local stress peaks and distributions really matter from a practical perspective, or could be considered as artefacts of the mathematical model, depends on the particular application under consideration. In any case, accurate stress predictions with evidence of convergence and knowledge of the stress peaks are prerequisites for reliable failure predictions of structures.

In this presentation, a DPG approach suitable for strength analysis of plate and shell structures is outlined. The underlying mathematical models are assumed to be of Kirchhoff-Love type, where the transverse shear stress resultants are defined in terms of the equilibrium equations only. The DPG approach is based on the trace theory developed in [1]. Recently, a formulation that can capture adaptively boundary and interior layers of curved shell deformations has been proposed in [2]. The presentation summarizes the most important theoretical findings together with numerical convergence studies for selected benchmark problems.

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DPG FOR REISSNER–MINDLIN PLATES, PART 2

THOMAS FÜHRER, NORBERT HEUER*, ANTTI H. NIEMI

ABSTRACT

The challenge of finding proper DPG settings for thin structure models consists in two parts: deriving a uniformly stable variational formulation and dealing with locking phenomena. In [3], we presented a variational formulation for the Kirchhoff–Love plate bending model, the abstract limit case of the Reissner–Mindlin model. The results in [3] apply to all physically relevant boundary conditions, and include non-convex Lipschitz plates. In [5], we extended this setting to the Reissner–Mindlin case, thus achieving a uniformly (with respect to the plate thickness) stable formulation with resulting quasi-optimal DPG scheme. These results were presented at the previous meeting, 2019 in Berlin. The question of appropriate discretization spaces and transverse shear-locking was open in the case of non-smooth solutions. In this talk we present a new formulation [4] that is based on a Helmholtz decomposition of the shear force variable, a technique proposed by Brezzi, Fortin [2] and thoroughly analyzed by Arnold, Falk [1] for a mixed finite element scheme. Our DPG scheme is provably locking free for convex hard-clamped plates.

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Sponsors

The MINRES/LS-5 workshop is sponsored by *Facultad de Matemáticas* of the *Pontificia Universidad Católica de Chile*

and the *Vicerrectoría de Investigación* of the *Pontificia Universidad Católica de Chile*.