TOWARDS ADAPTIVE HYBRID HIGH-ORDER METHODS (HHO)

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ABSTRACT

The novel methodology of skeletal schemes led to a new generation of nonstandard discretisations and HHO is one of many of those besides HDG, VEM, DPG, ... that generalize naturally to nonlinear problems. Can a variational crime lead to discretisations superior to conforming ones? The key for the success of higher-order schemes is through adaptive mesh-refining and the basis of this is a reliable and efficient a posteriori error analysis. The later is a topic in its infancy at least for HHO [5]. While over-stabilization enables some progress for DG and VEM, it is a refined analysis [6] that makes a stabilization-free a posteriori error estimate possible for the HHO [1]. The presentation reports on recent progress for linear problems [1] and then focusses on two very different nonlinear applications with — in comparison to conforming FEM — complementary advantages.

The fine-tuned extra-stabilized direct computation of guaranteed lower eigenvalue bounds allows for optimal convergence rates of a variant of HHO [2]. The appealing robust parameter selection allows the adaptive computation with higher convergence rates in numerical benchmarks.

The class of degenerate convex minimization problems with two-sided growth conditions and an appropriate convexity control [3, 4] allows convergent adaptive mesh-refinement for the dual stress-type variable.

The presentation reports on the state of research in joint projects with S. Puttkammer (Berlin) and N.T. Tran (Jena)

References

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