RIGOROUS GLOBAL MINIMIZATION OF NONLINEAR INTEGRAL FUNCTIONALS USING FINITE ELEMENT DISCRETIZATIONS AND POLYNOMIAL OPTIMIZATION

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ABSTRACT

Computation of minima of nonlinear integral functionals (e.g. strain energy) is typically done by using gradient descent methods or some version of Newton's method on the Euler-Lagrange PDEs associated with the functional. These procedures only guarantee finding an approximation to a local minimum, but say nothing of whether the solution is a global minimum of the functional, which is often the goal. Finding an algorithm that provably converges to a global minimum and corresponding minimizer is a classical and fundamental challenge in many fields, including nonlinear elasticity, fluid mechanics, pattern formation and PDE analysis. In this work, we leverage theoretical tools from the fields of sparse polynomial optimization (within algebraic geometry) and finite element (FE) methods to present such an algorithm. The techniques include exploiting properties of sparse sum-of-squares (SOS) relaxations and Gamma convergence to prove convergence to a global minimum of a functional with an integrand with polynomial nonlinearities as the mesh is refined and the moment-SOS relaxation order is raised. We present numerical examples which result in excellent approximations to the global minima of different nonlinear functionals, including the pattern-forming Swift-Hohenberg free energy in two spatial dimensions.

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