NEURAL CONTROL OF DISCRETE WEAK FORMULATIONS OF PDES

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ABSTRACT

We introduce the concept of neural control of discrete weak formulations of Partial Differential Equations (PDEs), in which finite element discretizations are intervened by using neural-network weight functions. The weight functions act as control variables that –through the minimization of a cost (or loss) functional– produce discrete solutions incorporating user-defined desirable attributes (e.g., known-data features, remotion of spurious oscillations, or precision at a certain quantities of interest).

Well-posedness and convergence of the cost-minimization problem are analyzed. In particular, we prove under certain conditions, that the discrete weak forms are stable, and that quasi-minimizing neural controls exist, which converge quasi-optimally. We specialize our analysis into Galerkin, least-squares, and minimal-residual formulations. Elementary numerical experiments support our findings and demonstrate the potential of the framework.

References

[1] Brevis, I., Muga, I. and van der Zee, K. G., *Neural Control of Discrete Weak Formulations: Galerkin, Least-Squares & Minimal-Residual Methods with Quasi-Optimal Weights*, arXiv:2206.07475 [math.NA] (2022).

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