APPLICATIONS OF A SPACE-TIME FOSLS FORMULATION FOR PARABOLIC PDES

GREGOR GANTNER*, ROB STEVENSON

ABSTRACT

While the common space-time variational formulation of a parabolic equation results in a bilinear form that is non-coercive, [1] recently proved well-posedness of a space-time first-order system least-squares (FOSLS) formulation of the heat equation, which corresponds to a symmetric and coercive bilinear form. In particular, the Galerkin approximation from any conforming trial space exists and is a quasi-best approximation. Additionally, the least-squares functional automatically provides a reliable and efficient error estimator. In [2], we have generalized the least-squares method of [1] to general second-order parabolic PDEs with possibly inhomogenoeus Dirichlet or Neumann boundary conditions. For homogeneous Dirichlet conditions, we present convergence of a standard adaptive finite element method driven by the least-squares estimator [2]. The convergence analysis is applicable to a wide range of least-squares formulations for other PDEs, answering a long-standing open question in the literature. Moreover, we employ the space-time least-squares method for parameter-dependent problems as well as optimal control problems [3]. In both cases, coercivity of the corresponding bilinear form plays a crucial role. Optimal control problems have been considered in parallel by [4].

References

[1] T. Führer and M. Karkulik., *Space–time least-squares finite elements for parabolic equations*, Comput. Math. Appl. 92 (2021), 27–36.

[2] G. Gantner and R. Stevenson, *Further results on a space-time FOSLS formulation of parabolic PDEs*, ESAIM Math. Model. Numer. Anal. 55.1 (2021), 283–299.

[3] G. Gantner and R. Stevenson, *Applications of a space-time FOSLS formulation for parabolic PDEs*, Preprint, arXiv:2208.09616 (2022).

[4] T. Führer and M. Karkulik., *Space-time finite element methods for parabolic distributed optimal control problems*, Preprint, arXiv:2208.09879 (2022).

* Institute of Analysis and Scientific Computing, TU Wien, Austria, gregor.gantner@asc.tuwien.ac.at